

Advanced Mathematics Teaching Reform in the Generative AI Era

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Abstract: This paper proposes an integrated reform model for Advanced Mathematics based on outcome-based education (OBE), problem-based learning (PBL), blended learning, case teaching, and human-led use of generative AI. The model, termed DICE-OBE, organizes teaching into four stages: Diagnose, Inquire, Construct, and Evaluate. It combines prerequisite diagnosis before class, problem and case triggers during class, rigorous mathematical construction, and evidence-based post-class evaluation. The central principle is that digital tools should increase opportunities for mathematical thinking without replacing mathematical judgment, proof, or communication.

Keywords: Advanced Mathematics; Teaching reform; Generative artificial intelligence; OBE; Problem-based learning; Blended learning

Online publication: October 20, 2026

1. Introduction

Advanced Mathematics is a foundational course for science, engineering, economics, management, and related programs. Its educational purpose is broader than symbolic manipulation: students need to understand definitions, reason from assumptions, construct proofs or derivations, model quantitative situations, and transfer concepts to unfamiliar problems. However, conventional lecture-centered instruction often separates concepts from applications and gives students limited time to explain, compare, or critique mathematical reasoning.

At the same time, online platforms, dynamic visualization, computer algebra systems, and generative AI can provide rapid feedback, alternative explanations, examples, and computational checks. The pedagogical question is therefore not whether technology should enter the course, but how it should be organized so that technology strengthens rather than substitutes for mathematical thinking.

Three instructional tensions are particularly important. The first is the gap between procedural fluency and conceptual understanding: students may execute differentiation or integration algorithms while remaining

uncertain about the meaning of a limit, the role of a hypothesis, or the interpretation of an accumulated quantity. The second is the gap between classroom coverage and individual learning pace: a single lecture sequence can move too quickly for students with weak prerequisites while offering too little challenge to students who have already mastered routine procedures. The third is the gap between answer production and mathematical communication: digital systems can make an answer easy to obtain, but university-level mathematics requires students to state definitions precisely, justify transitions, interpret results, and recognize when a method is inapplicable.

A review of recent publications in the target journal shows recurring interest in blended mathematics teaching, OBE/PBL, case teaching in Higher Mathematics, digital-intelligence-enabled instruction, teacher competence in the AI era, and personalized learning ^[1-7]. These strands suggest a useful synthesis: outcome alignment can provide the course architecture, cases and problems can organize classroom activity, blended learning can move routine preparation online, and AI can be used selectively for diagnosis, variation, feedback, and verification.

The proposed reform therefore treats these approaches as complementary rather than competing. OBE answers the question of what students should be able to demonstrate ^[8]; PBL and case teaching answer how a mathematical need can be made visible ^[9]; blended learning and active-learning structures support productive classroom participation ^[10]; and digital tools provide rapid representations and feedback, consistent with the broader discussion of AI in higher education ^[11]. The missing link in many implementations is a mechanism that tells teachers when to move from exploration to formalization and how to preserve independent reasoning when AI is available. DICE-OBE is designed to supply that mechanism at the level of a weekly teaching cycle.

2. Reform framework

2.1. Design principles

The reform follows four principles. First, outcomes must be observable and aligned with learning activities and assessment. Second, students should encounter mathematical ideas through problems that create a need for the concept before formalization. Third, online and classroom work should have different functions: routine retrieval and preparation can be automated, while proof, interpretation, discussion, and transfer remain central classroom tasks. Fourth, AI assistance should be transparent and subordinate to mathematical validation: students must be able to explain why a solution is correct, identify errors, and produce independent work.

2.2. The DICE-OBE model

DICE-OBE denotes Diagnose, Inquire, Construct, and Evaluate (**Table 1**). OBE supplies the alignment logic; PBL and case teaching supply the learning triggers; blended learning supplies the workflow; visualization and AI supply optional tools. The four stages recur within each unit rather than representing a one-time project.

The value of the cycle lies in the transitions between stages. Diagnosis makes prerequisite gaps visible before new content is introduced. Inquiry creates a cognitive reason to learn the new concept rather than presenting it as an isolated formula. Construction restores the rigor that exploratory activity can omit. Evaluation then checks whether students can transfer the constructed knowledge to a new setting. In this

sense, DICE-OBE is not simply a four-step lesson plan; it is an alignment mechanism linking learning evidence to instructional decisions.

Table 1. The DICE-OBE model

Stage	Core question	Typical activities	Digital/AI role
Diagnose	What is missing?	Prerequisite retrieval; misconception check; individual goal setting.	Analytics and adaptive item banks; optional AI explanation of prerequisite ideas.
Inquire	Why is the new mathematics needed?	Case/problem trigger; estimation; conjecture; group discussion.	Graphs, simulation, dynamic visualization; generation of comparable examples.
Construct	How is intuition made rigorous?	Definition; theorem conditions; proof/derivation; method comparison; guided practice.	CAS/Python for checking; AI as a candidate solution to critique, not as authority.
Evaluate	What can be explained, solved, modeled, and transferred?	Individual problem solving; modeling; reflection; revision.	Automated feedback for routine items; dashboards for learning evidence.

3. Curriculum and teaching design

Course content should be reconstructed in three layers. Layer 1 is the mathematical core: precise definitions, essential theorems, proof structures, standard methods, and typical counterexamples. Layer 2 contains transfer tasks: modeling problems, non-routine exercises, interpretation questions, and case activities that require students to select and combine concepts. Layer 3 provides extension resources such as visualization, computation, enrichment problems, and optional AI-supported exploration. This structure prevents technology from displacing the conceptual core while making room for differentiation.

A second reconstruction concerns the sequence of representation. Mathematical ideas should ordinarily appear in at least three forms: an intuitive or contextual representation, a formal symbolic representation, and an interpretive representation that explains what the result means. For example, a derivative may be introduced through local change, formalized as a limit of difference quotients, and then interpreted as a rate in a model. Moving repeatedly among these representations helps students avoid identifying a concept solely with one notation or algorithm.

Each unit should have a concept map and a task map. The concept map records prerequisites, central ideas, theorem dependencies, and common misconceptions. The task map records one trigger problem, one or two key construction tasks, a transfer task, and an individual assessment task. The weekly rhythm is therefore: pre-class retrieval and diagnosis; in-class inquiry; rigorous construction; guided and independent practice; and post-class evaluation.

Teacher and student roles should also be differentiated deliberately. Before class, students are expected to retrieve prerequisites and record unresolved questions; the teacher uses the resulting evidence to adjust the entry point. During class, the teacher organizes mathematical discourse, selects representative errors, and makes theorem conditions explicit, while students compare strategies and defend claims. After class, students revise mistakes and complete an independent transfer task. This division reduces the tendency for blended learning to become simply a lecture plus homework uploaded to a platform.

4. Example: Definite integral unit

4.1. Learning outcomes

Students should be able to (1) interpret the definite integral as an accumulation defined by a limiting process; (2) construct and interpret Riemann sums; (3) use the Fundamental Theorem of Calculus appropriately; (4) distinguish signed accumulation from geometric area; and (5) build and explain a simple integral model for an applied situation.

4.2. DICE implementation

Diagnose: Before class, students complete a short quiz on functions, partitions, sums, limits, and antiderivatives. Errors are grouped by concept rather than only by score. Students receiving low prerequisite feedback complete targeted retrieval activities.

Inquire: The teacher introduces a changing-speed motion problem. Students estimate distance using rectangles and discuss how partition width affects the approximation. The key question is formulated explicitly: what mathematical object represents the limiting accumulated quantity?

Construct: The class formalizes the partition, Riemann sum, and limiting definition. The teacher then connects the definition to the Fundamental Theorem of Calculus and compares representations: accumulation, signed area, and antiderivative. Students explain assumptions and conditions rather than merely copying formulas.

Verify with AI: Groups receive an AI-generated solution containing a subtle error, such as treating signed area as total area or applying a theorem without checking its conditions. Students annotate the response, identify the exact invalid step, provide a corrected argument, and verify the result independently. The pedagogical target is mathematical source criticism, not AI skill for its own sake.

Evaluate: The post-class task includes one conceptual question, one calculation problem, one modeling problem, and a brief reflection on a corrected misconception. One item is completed individually without AI assistance so that independent transfer remains observable.

A practical 90-minute lesson can be organized as follows. In the first 10 minutes, the teacher reviews diagnostic errors and asks students to predict whether a coarse partition will overestimate or underestimate the distance traveled. During the next 20 minutes, groups construct rectangle approximations using two or more partition choices and explain the effect of mesh size. The following 25 minutes are devoted to formalizing the Riemann sum and its limiting interpretation. In the next 15 minutes, students compare the definite integral with geometric area and identify cases in which signs matter. A 10-minute AI-verification activity then presents a flawed solution for critique, followed by 10 minutes of individual transfer work. The final minutes are used to collect a one-sentence conceptual exit response. The sequence is intentionally asymmetric: technology-supported exploration is brief, while formal reasoning and independent work occupy the central portion of the lesson.

4.3. Differentiation and learning support

The same unit can support different levels of prior preparation without splitting the class into unrelated tracks. Students who have difficulty with partitions and limits receive a short prerequisite module with worked examples and additional graphical representations. Students who meet the baseline can proceed directly to the modeling task. More advanced students can investigate nonuniform partitions, numerical

approximation, or the relationship between integration and accumulated error. The common core remains unchanged: every student must be able to state the definition, explain the limiting idea, use the Fundamental Theorem under appropriate conditions, and interpret the result. Differentiation therefore changes the route and degree of challenge, not the mathematical standards.

5. Assessment reform

Assessment reform should make the intended learning outcomes visible in the evidence collected. If a course claims that students should reason, model, communicate, and transfer, an assessment system based almost entirely on short symbolic calculations is structurally misaligned. The recommended approach is therefore longitudinal: small diagnostic tasks establish readiness; classroom tasks capture reasoning; case or modeling tasks reveal transfer; and individual assessments establish independent performance (**Table 2**). The purpose of repeated low-stakes evidence is not to increase testing frequency, but to give teachers and students actionable information before misunderstandings become cumulative.

Table 2. Assessment reform

Evidence source	Assessed competence	Feedback	Risk controlled
Online retrieval	Prerequisite fluency and recall	Automated item-level feedback	Unnoticed prerequisite gaps
In-class reasoning	Conjecture, proof steps, explanation, critique	Teacher rubric + verbal feedback	Passive participation
Case/modeling task	Model construction and interpretation	Rubric + revision memo	Formula-only problem solving
AI verification	Mathematical validation and evidence use	Annotated feedback	Uncritical AI dependence
Individual assessment	Independent transfer and communication	Error analysis	Outsourcing assessed work

Assessment should therefore combine low-stakes retrieval, reasoning evidence, written mathematical work, modeling or case tasks, and individual transfer. Rubrics should distinguish conceptual correctness, reasoning quality, mathematical communication, interpretation, and appropriate tool use. Students should disclose when AI is used and identify what they independently verified. This shifts assessment from product-only scoring toward evidence of learning processes and mathematical judgment.

6. Assessment criteria for AI-supported work

AI-supported work also requires a distinction between assistance and substitution. Assistance may include generating additional examples, comparing alternative explanations, checking an intermediate calculation, or suggesting questions for further study. Substitution occurs when a student submits machine-generated reasoning that the student cannot reconstruct or defend. A workable policy is therefore to assess provenance as part of mathematical communication: students indicate what tool was used, identify the portions they adopted or rejected, and provide an independent verification. This approach does not prohibit technology; it makes accountability explicit.

7. Implementation and evaluation

7.1. Implementation

Successful implementation requires teacher preparation as well as student preparation. Course teams should define approved and prohibited AI uses, prepare common rubrics, build a small bank of diagnostic items and cases, and establish routines for checking AI-generated content. Teachers also need enough digital fluency to interpret learning data and enough mathematical oversight to reject plausible but incorrect machine outputs.

Implementation is more manageable when reform is staged. During an initial pilot, a teaching team can redesign two or three high-leverage units and use a shared template for outcomes, prerequisite diagnosis, trigger problems, construction tasks, and transfer assessment. The second stage can examine student work and teacher observations to identify which activities genuinely improve reasoning and which merely add platform workload. The third stage can scale the validated components across additional units. Such staged adoption also allows teachers to build a local repository of cases and common misconceptions rather than repeatedly creating materials from scratch.

The reform can be evaluated with a mixed-method design. A feasible quasi-experimental study can compare a DICE-OBE class with a conventional class using pre- and post-tests that separately measure conceptual understanding, procedural fluency, modeling, and transfer. Classroom observations, student work, short interviews, and learning analytics can explain why outcomes change. Reporting should include effect sizes and uncertainty estimates when sample size permits, rather than relying only on satisfaction surveys. A four-step cycle—baseline diagnosis, implementation monitoring, post-unit measurement, and revision—allows the teaching model to improve iteratively.

7.2. Evaluation indicators

Evaluation should distinguish at least four dimensions. Conceptual understanding concerns definitions, relationships, and explanations. Procedural fluency concerns accurate execution of standard methods. Transfer concerns the selection and combination of mathematical ideas in unfamiliar situations. Metacognitive and communication outcomes concern students' ability to identify uncertainty, explain why an argument works, and revise an erroneous solution. Learning analytics can document participation and completion, but these process indicators should not be interpreted as direct evidence of mathematical learning. The strongest evidence comes from triangulating task performance, written work, observation, and student reflection.

8. Discussion and conclusion

The proposed reform does not treat digitalization as an end in itself. Its distinctive feature is the explicit division of pedagogical labor: technology handles routine retrieval, variation, visualization, and some forms of feedback; teachers remain responsible for mathematical structure and judgment; students remain responsible for reasoning, explanation, verification, and independent transfer. This division is especially important in Advanced Mathematics, where correct answers without valid reasoning do not demonstrate full learning.

The model is also consistent with a broader principle of instructional economy. A technology-supported activity deserves a place in the course only when it creates learning value that is difficult to obtain as efficiently through conventional means. A dynamic visualization may clarify a changing quantity; an adaptive

quiz may reveal a prerequisite gap within minutes; an AI-generated counterexample may create an efficient opportunity for critique. By contrast, asking students to use AI simply to produce routine solutions can reduce rather than increase mathematical engagement. The criterion for adoption should therefore be pedagogical function, not novelty.

The model also has practical constraints. Developing cases, diagnostic resources, and rubrics requires initial investment, and AI-supported activities can fail when prompts are weak or generated mathematics is inaccurate. These limitations support gradual implementation rather than wholesale redesign. A small number of high-value units—such as limits, definite integrals, sequences and series, or multivariable optimization—can serve as pilots before semester-wide adoption.

There are also research limitations in the proposed framework. Without controlled implementation, it is not possible to infer that the model itself causes improvement, and satisfaction or usage data should not be treated as substitutes for achievement measures. Differences in instructor expertise, student preparation, course level, and access to technology may moderate the results. Future studies should therefore report the instructional context clearly and, where feasible, use comparable pre- and post-assessments with transparent scoring procedures. Replication across institutions and student populations would strengthen the external validity of any observed effects.

In conclusion, Advanced Mathematics teaching can be reformed by aligning outcomes, problems, rigorous mathematical construction, blended learning, and responsible AI use within one repeatable cycle. DICE-OBE offers a compact framework for doing so and provides a concrete basis for empirical research on whether digitally supported teaching can produce deeper mathematical understanding without reducing students' independence or rigor. Its practical contribution is to turn broad reform principles into a repeatable unit-level process with explicit roles for diagnosis, inquiry, construction, evaluation, and responsible tool use.

Funding

This research was supported by the Fundamental Research Funds for the Central Universities (Grant No. 2652025043).

Disclosure statement

The author declares no conflict of interest.

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