

Synthesized Dual-Resonance Model for Revealing the Isotropic Uniform Symmetry (IUS) of Planetary Orbits

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Abstract: Planetary Perturbations are caused by multi-body gravity and relativity. The first kind of perturbation method is to use a high-order numerical approximation or analytical series to obtain a result. Based on the SDP atom model, the study has developed a Synthesized Dual-Resonance model to describe orbital precession as the coupling of radial resonance and circumferential motion. The model has an integral form for angular momentum. Numerical simulations have shown that the shape of the orbit and its convergence properties at all frequencies are different, and cumulative orbits approach an isotropic, uniform, symmetric ring around a single dynamical center in the long-period limit. It has a relatively simple kinematic structure for quick orbit design and long-period prediction.

Keywords: Spiral Drift Precession Model (SDP); Isotropic Uniform Symmetry (IUS); Sole centrism of a stellar system; Closures of planetary orbits

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1. Introduction

After Kepler proposed the three laws of planetary motion, Newton derived the universal law of gravitation and built a unified dynamical system^[1-3]. With more precise observations of multi-body motion and general relativity, it has been determined that the Keplerian model is not accurate. Tycho Brahe had limited observation space and time, and Newton's inverse-square law is an approximation of the two-body problem.

Laplace, Lagrange, and Poincaré also had these shortcomings^[4-6]. Poincaré showed that there is no general analytic solution to the three-body problem and introduced chaos^[6]. In the early 20th century, Einstein's general relativity explained Mercury's 43"/century perihelion precession^[7]. At present, the two main ways of predicting orbits are high-precision numerical integration (such as Cowell's method and Encke's method) and analytical perturbation theory^[8-10]. However, numerical methods have efficiency

problems in long-term development, and although analytic theories are physically intuitive, their complicated formulas are not very intuitive geometrically ^[11–12].

Based on Newton's study of spiral motion in a resistive medium and the team's SDP atomic model (which shows radial resonances), the authors propose that orbital precession can be considered the sum of independent radial and azimuthal oscillatory motions ^[13, 15–18]. A parametric dual-resonance model is constructed, angular momentum is conserved, and it can be shown that the long-term cumulative orbit converges to an isotropic uniform ring; thus, the idea of a single dynamical center is supported ^[14, 19].

2. A kinematics-based dual-resonance model

Based on Newton's idea of spiral motion in a resisting medium and the radial resonance in the SDP atomic model, the authors use a low-viscosity medium to model the gravitational field ^[13, 15–18]. Both the gravitational field and the Coulomb field are conservative, so they are analogous. In weak fields, the multi-body perturbation is in the form of oscillations rather than inspirals; thus, the authors assume independent periodic radial perturbations and develop the Dual-Resonance model accordingly.

$$\begin{aligned} \mathbf{r}(t) &= \begin{pmatrix} r(t)\cos[\varphi(t)] \\ r(t)\sin[\varphi(t)] \end{pmatrix} \\ r(t) &= r_0 + a\cos(\omega_r t) \\ \varphi(t) &= \omega_\varphi t \end{aligned} \quad (1)$$

Where $\omega_r = \frac{2\pi n}{T_r}$, $\omega_\varphi = \frac{2\pi n}{T_\varphi}$, and r_0 denotes the initial radius, a represents a perturbation much smaller than r_0 , while ω_r and $\varphi(t)$ correspond to describe the radial motion frequency and the circumferential motion equation, respectively.

Angular momentum is conserved in a conservative field, so the azimuthal speed will vary with the radius. The corrected equation is:

$$\begin{aligned} \mathbf{r}(t) &= \begin{pmatrix} r(t)\cos[\varphi(t)] \\ r(t)\sin[\varphi(t)] \end{pmatrix} \\ r(t) &= r_0 + a\cos(\omega_r t) \\ \varphi(t) &= \frac{2\pi n}{T_\varphi} \int_0^t \frac{r_0^2}{r(\tau)^2} d\tau \end{aligned} \quad (2)$$

This defines the Trigonometric Function-Based Model (TFBM). It couples radial and angular motions, producing precession naturally when m/n is non-integer or when angular momentum correction shifts the phase advance per cycle.

3. Definitions of convergence and isotropic uniform symmetry

In actual observations, to determine whether an orbit is closed, one needs to examine the data from observations and then judge whether it is closed. There is an error here: since the closure conditions of real orbits may not strictly meet the ratio of rational numbers, an orbit relatively closed or isotropic uniform can be considered if the observed values fall within the expected range. An orbital density function and a convergence function are introduced here to describe the density and uniformity of orbits in a given spacetime, respectively. The convergence function has reached a steady state, and the orbits will be relatively uniform.

Trajectory Density (TD) is a spacetime-related function $TD(\varphi, r, t)$:

Given the sampling azimuth angle $\varphi = \varphi_n$, $n=1, 2, \dots, N$ (e.g., taking one φ every 10 degrees yields $N=36$), and the number of circumferential turns $n=1, 2, \dots, M$, the spacing (or pitch, taken as absolute value) between the adjacent tracks of the $(m-1)$ -th and m -th turns of the circumferential spiral motion (calculated from the track equation) is:

$$\Delta r_{m,n} = |r_{m,n} - r_{m-1,n}| = |r_m(\varphi_n + 2\pi) - r_{m-1}(\varphi_n)| \quad (3)$$

The orbital density at coordinates (m, n) is defined by the quotient of the drift bandwidth $|r_{\max} - r_{\min}|$ and the orbital spacing (where the drift bandwidth in the model is twice the amplitude, i.e., $2a$):

$$TD(m, n) = \frac{|r_{\max} - r_{\min}|}{|\Delta r_{m,n}|} = \frac{2a}{|\Delta r_{m,n}|} \quad (4)$$

The smaller the distance between adjacent tracks, the smaller the pitch, the tighter the tracks, and the greater the density. After calculations are complete but before plotting, TD is normalized by the maximum density value:

$$\overline{TD}(m, n) = \frac{TD(m, n)}{\max_{n \leq n' \leq N} TD(m, n')} \quad (5)$$

For a given azimuth angle $\varphi = \varphi_n$, where $n=1, 2, \dots, N$ (e.g., selecting φ every 10 degrees yields $N=36$), compute and plot the pattern curve of orbital density values for all revolutions ($m=1, 2, \dots, M$) (plotting the computational simulation diagram: independent variable $\sim t$):

$$\overline{TD}(m, n) \sim t \quad (6)$$

To demonstrate that the orbit is isotropic and converges in density after the M -th revolution, the convergence function CV is defined using the standard deviation at a given revolution m and azimuth n . This serves as a measure of intra-revolution uniformity for the orbit density at the m -th revolution. Let:

$$v(m, n) = \frac{1}{m} \sum_{n=1}^m \overline{TD}(m, n) \quad (7)$$

$$\mu(m) = \frac{1}{N} \sum_{n=1}^N v(m, n) \quad (8)$$

Then CV is defined as:

$$CV(m) = \frac{1}{N} \left\{ \sum_{n=1}^N [v(m, n) - \mu(m)]^2 \right\}^{0.5} \quad (9)$$

Normalize to the maximum value:

$$\overline{CV(m)} = \frac{CV(m)}{\max_{m' \leq m} CV(m')} \quad (10)$$

Convergence Principle:

The convergence function $CV(m)$ measures the dispersion of orbital density across azimuths in the m -th orbit. As $CV(m)$ approaches stability, orbital density becomes uniform across azimuths. Additionally, as long-period orbits converge, the cumulative orbit exhibits an isotropic uniform ring-band structure, with a cumulative trajectory being isotropically symmetric. Here, an Isotropic Uniform Symmetry (IUS) function is constructed to assess the isotropic uniformity and symmetry of cumulative orbits at different time points:

$$IUS(m) = \left(\frac{1 - [CV(m)]^2}{1 + [CV(m)]^2} \right)^{0.5} \times 100\% \quad (11)$$

When the value of IUS reaches 100%, it indicates that the orbit approaches isotropic uniformity and symmetry.

4. Orbit simulation and numerical analysis

The first kind of the double-resonance model is to combine the radial resonance frequency and the circumferential motion frequency. Therefore, one should examine the orbit and stability of different frequency ratios. TFBM, the model introduced in the previous section, will be discussed here. Based on different frequency ratios of TFBM, the shape of the orbit will be studied. Finally, the orbital figures have shown the characteristics of both circumferential standing waves and radial standing waves, and thus the dual-resonance model can describe many types of orbital modes. The convergence results show that the planetary orbits are relatively regular over a long period, which is in line with the idea of a single dynamical center.

4.1. Visualization of orbital patterns in TFBM

Based on the dual-resonance motion synthesis characteristics of TFBM and the corresponding changes in circumferential motion frequency due to angular momentum conservation, the orbital patterns of the model will naturally be different at various frequency ratios. When the frequency ratio is an integer multiple, then there will be radial resonance and circumferential period standing wave orbits with precession, as shown below.

Let $\omega_r/\omega_\phi = \eta$ denote the ratio of the radial frequency to the circumferential frequency. The results are presented in a planar phase projection diagram. The frequency ratios selected are $\eta < 1$, $\eta = 1$, and $\eta > 1$. The orbital characteristics are most pronounced at the representative frequency ratio. The orbital patterns generated by these three sets of resonance frequencies are more suitable for comparing differences, as other orbits are either too dense or too sparse, resulting in indistinct vortices or insufficiently pronounced oscillation smoothness.

By analyzing orbital images with different frequency ratios, a pronounced precession effect is observed in TFBM. Under the condition of $\eta < 1$, the trajectory with the most characteristic orbital frequency ratio is presented in **Figure 1**. Here, the horizontal X-axis and vertical Y-axis represent the two-dimensional plane of

the actual planetary orbital space. The orbital radius has been normalized relative to r_0 , with both X and Y coordinates ranging from (-1, 1). The solid line depicts the short-period cumulative trajectory of a perturbation orbit with an amplitude of $0.1r_0$, obtained when the ratio of radial to orbital frequencies is 1:3 and corrected for angular momentum effects. The square at coordinates (0,0) indicates the orbital center point, representing the star's position, while the sphere denotes the planet's spatial location:

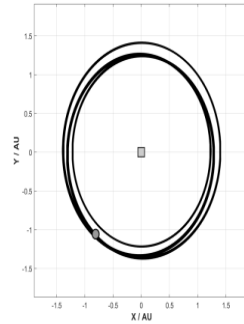


Figure 1. Orbital figure of $\eta=1/3$

Under the condition of $\eta > 1$, select the frequency ratio with the most pronounced orbital characteristics, $\eta=6$, as presented in **Figure 2**.

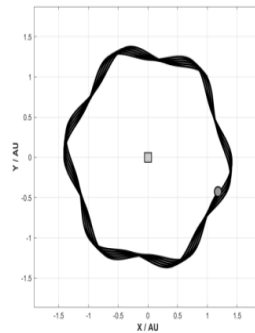


Figure 2. Orbital figure of $\eta=6$

Given the combination of the two frequency ratios, the orbit of TFBM will have both radial and circumferential standing-wave patterns and a large precession. Add an extra net angular increment after one radial or circumferential period; otherwise, it will close, and there will be no precession.

4.2. Convergence analysis

Section 3 has introduced the definition of the convergence function. The following is the convergence behavior of TFBM under a certain frequency ratio after computer simulations. It will also compare and contrast the effect of angular momentum correction on the model's orbit. For clarity, the convergence functions of the most symbolic frequency ratio are chosen for illustration in **Figure 3**.

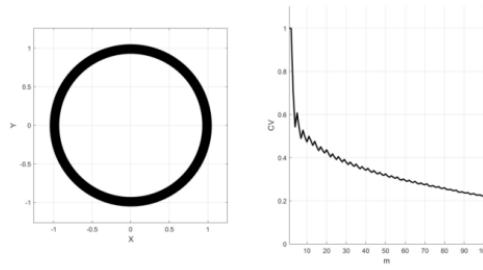


Figure 3. The left side displays the isotropic characteristics of the long-period cumulative orbit. The right side depicts the convergence function of the orbit as a function of the number of revolutions m on the horizontal axis and CV on the vertical axis. This image illustrates the behavior at a frequency ratio $\eta=1/3$. It is evident that the overall CV trend steadily stabilizes and converges as the number of revolutions increases, indicating that the orbit ultimately tends toward uniformity and stability

Based on the convergence analysis result of TFBM, the CV value approaches a constant convergence value during long-term operation and ultimately stabilizes at a steady state with minimal fluctuations. This indicates that the dual-resonance model achieves convergence under long-term conditions, demonstrating the model's high computability and stability.

4.3. Show the change process of isotropic uniform symmetry (IUS)

In Section 3, the isotropic symmetry functions for the IUS were constructed. This section presents the cumulative orbital isotropic uniformity of TFBM at a specific frequency ratio. Furthermore, computer simulations will be used to analyze the isotropic symmetry characteristics of planetary cumulative orbits. For a typical computation case $\eta=1/3$, the Isotropic Uniform Symmetry (IUS) is displaying dynamically from **Figures 4–6**.

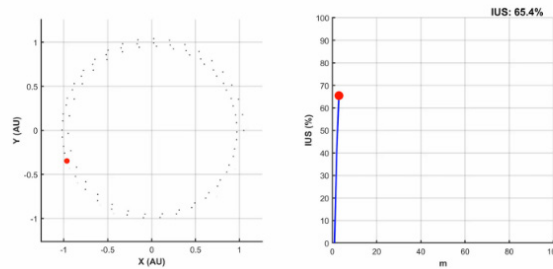


Figure 4. The horizontal axis m and vertical axis IUS describe the trend of the orbital convergence function as the number of revolutions m changes. This figure illustrates the IUS value of TFBM with a frequency ratio $\eta=1/3$ is 65.4% in short periods

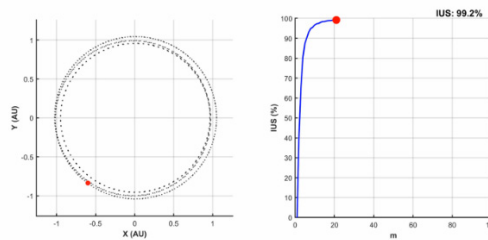


Figure 5. The IUS value of TFBM with a frequency ratio $\eta=1/3$ is 99.2% after more periods

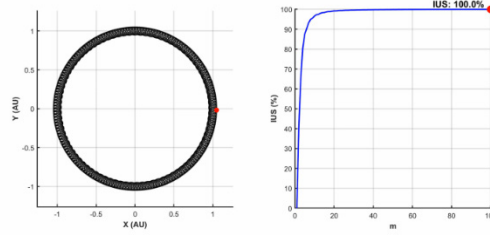


Figure 6. It shows that IUS reaches 100% under long-term condition

Therefore, it can be concluded that the long-term dynamical evolution of the accumulated orbit is isotropic, uniform, and symmetric, and there will be a dynamical balance of perturbations in multi-body systems. The perturbed orbits given by the model are always periodic and revolve around the same center; thus, they still conform to the idea that planetary orbits have a single dynamical center.

4.4. Calculation and presentation of a real case

To evaluate the behavior of the Dual-Resonance model at physical scales, the authors selected the orbit of Mars for a full-scale simulation. Mars' eccentricity, $e = 0.0934$, is relatively low and approximates a circular orbit, making it suitable for use as input data in the Dual-Resonance model. Model parameters were assigned strictly according to their kinematic definitions, based on actual observational data: the mean radius was taken as the semi-major axis $r_0 = 1.523679 AU$. The radial amplitude is defined here as the magnitude of variation in the orbital radial direction; for a nearly circular orbit, this can be regarded as the eccentricity, so a was taken as $a = 0.0934 \times r_0$. The trajectory and IUS results using Mars' orbital parameters are displayed in **Figures 7 and 8**.

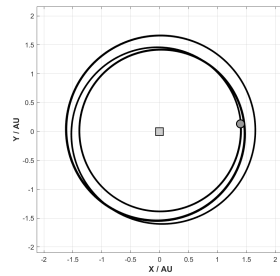


Figure 7. Orbital Figure using Mars' parameters with $\eta=1/3$

Here is the frequency ratio that achieves the best convergence in the IUS; substitute this value into the Mars orbital parameters and calculate the IUS.

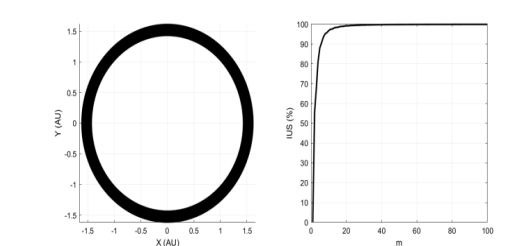


Figure 8. It shows that the IUS of Mars' cumulative orbit reaches 100% under long-term condition.

Based on the simulation results, substituting the actual orbital parameters also leads to the same conclusion for IUS; it still converges in the long run, and the cumulative orbit has the form of an isotropic orbital belt. It can be seen that the double-resonance model has quasi-periodic motion in real life. Thus, it provides an intuitive kinematic analog of the KAM surface in planetary orbits and shows that the model is also a phenomenological model^[19].

5. Conclusion

This study constructs a physically self-consistent and mathematically rigorous kinematic-based dual-resonance model, the Trigonometric Function-Based Model constrained by angular momentum conservation for specific cases. The core conclusion is that, in the long-period limit, cumulative orbits will form an isotropic, uniform, symmetric orbital ring with a single dynamical center.

The long-term isotropic uniform symmetry of this model is verified by constructing orbital density functions and convergence functions. In terms of applied values, the model offers a new perspective for understanding complex orbital phenomena from the angles of motion synthesis and kinematic modeling. It predicts the existence of periodic perturbations in the radial direction of planetary orbits and establishes the foundation for an expandable, rapid orbital phenomenological analysis framework. Theoretically, it enhances and refines Kepler's laws and Newton's law of universal gravitation, treated as zero-order approximations, from a kinematic perspective. To broaden the application values of this dual-resonance model, the authors plan future enhancements including elliptical orbit corrections, perturbation source induction, general relativity field equation modifications, and computational efficiency comparisons with traditional analytical perturbation and numerical integration methods. This will provide precise and effective kinematic solutions for preliminary space mission design and orbital simulation.

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Disclosure statement

The authors declare no conflict of interest.

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